Nonlinear Transient and Steady State Stretching of Deflated Vesicles in Flow

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ABSTRACT: Membrane-bound vesicles and organelles exhibit a wide array of nonspherical shapes at equilibrium, including biconcave and tubular morphologies. Despite recent progress, the stretching dynamics of deflated vesicles is not fully understood, particularly far from equilibrium where complex nonspherical shapes undergo large deformations in flow. Here, we directly observe the transient and steady-state nonlinear stretching dynamics of deflated vesicles in extensional flow using a Stokes trap. Automated flow control is used to observe vesicle dynamics over a wide range of flow rates, shape anisotropy, and viscosity contrast. Our results show that deflated vesicle membranes stretch into highly deformed shapes in flow above a critical capillary number $C_{ac}$, We further identify a second critical capillary number $C_{ac2}$ above which vesicle stretch diverges in flow. Vesicles are robust to multiple nonlinear stretch−relax cycles, evidenced by relaxation of dumbbell-shaped vesicles containing thin lipid tethers following flow cessation. An analytical model is developed for vesicle deformation in flow, which enables comparison of nonlinear steady-state stretching results with theories for different reduced volumes. Our results show that the model captures the steady-state stretching of moderately deflated vesicles; however, it underpredicts the steady-state nonlinear stretching of highly deflated vesicles. Overall, these results provide a new understanding of the nonlinear stretching dynamics and membrane mechanics of deflated vesicles in flow.

INTRODUCTION

Membrane-bound vesicles and cells with nonspherical shapes are commonly found in biology. Red blood cells (RBCs) exhibit biconcave disc shapes at equilibrium that increase surface area to enhance oxygen transport across the membrane. RBCs also undergo repeated, large-amplitude mechanical deformations while flowing through microvessels. From this view, there is a clear need to understand the nonequilibrium dynamics and mechanics of membrane-bound vesicles with tubular or biconcave morphologies, including the role of membrane deformability, which will reveal new insight into the mechanics and transport properties of cells. Prior work on vesicle mechanics has largely focused on nearly spherical vesicle morphologies in the weak deformation regime using micropipette aspiration, electrodeformation, optical trapping, and sedimentation. In recent years, vesicle dynamics have been studied in defined hydrodynamic flows including capillary flow, simple shear, pure extension, and linear mixed flows. Vesicle morphology in flow depends on flow strength and shape anisotropy. The reduced volume $\nu$ is a measure of vesicle shape anisotropy, such that $\nu = 1$ corresponds to a perfect sphere. In Poiseuille flow, weakly deflated vesicles ($\nu > 0.7$) adopt an axisymmetric parachute shape with a concave terminus or a bullet-like shape with a convex terminus depending on flow strength. For highly deflated vesicles ($\nu < 0.7$), a slipper-like shape is observed over a wide range of flow rates. In shear flow, vesicles undergo three different dynamical motions including: (i) tumbling (TU) motion in which the vesicle exhibits full periodic rotations ($\phi = 2\pi$) with respect to the flow-axis, (ii) tank-treading (TT) motion in which an ellipsoidal vesicle’s major axis maintains a constant angle with respect to the flow-axis and the membrane rotates around this fixed ellipsoidal shape, and (iii) trembling (TR) motion in which the vesicle’s major axis oscillates around the flow-axis but never reaches $\phi = \pi/2$. Classic work by Keller and Skalak for ellipsoidal-shaped vesicles with negligible thermal fluctuations theoretically predicted tumbling and tank-treading dynamics, which was experimentally observed several years later by Kantsler and co-workers in 2006. Theoretical modeling by Misbah and co-workers revealed the trembling behavior of vesicles in shear...

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and an analytical theory by Vlahovska and co-workers was also developed to describe the dynamics of a freely suspended lipid bilayer vesicle in a general linear flow. A phase diagram representing the transitions between the three dynamical regimes in shear flow was achieved by varying the viscosity contrast between the vesicle interior and exterior and eventually for a general linear flow. Moreover, an analytical model was further developed to describe transitions between these different dynamical states in a 2D external flow. Zhao and Shaqfeh determined the critical viscosity ratios for the transition from TT to TR to TU regimes using a linear stability analysis based on a spectral boundary integral method, which showed good agreement with experiments. Vesicle dynamics in shear flow has been further studied to include the large deformation of vesicles in the presence of soluble surfactants and role of thermal fluctuation in membrane shape dynamics. In extensional flow, highly deformed vesicles with tubular morphologies ($\nu < 0.6$) transition to symmetric dumbbells above a critical flow strength, whereas moderately deformed vesicles with biconcave and spheroidal morphologies ($0.6 \leq \nu < 0.75$) transition to asymmetric dumbbell shapes in flow. A flow-phase diagram for vesicles in extensional flow was recently determined across a wide range of reduced volumes, revealing vesicle morphology in the vicinity of the critical flow strength ($\dot{\epsilon}_c$) for morphological changes. However, the transient stretching behavior of membrane-bound vesicles above the critical strain rate $\dot{\epsilon}_c$ has not been systematically characterized. Recent work has focused on the conformational relaxation of highly deformed deformed vesicles ($\nu < 0.75$) following nonlinear deformation into a dumbbell shape in flow. Interestingly, it was found that vesicle relaxation is governed by a double-mode process, with the first stage involving relaxation of the thin lipid nanotube connecting the two spherical bulbs in the dumbbell. Vesicle dynamics in a time-dependent extensional flow with a single flow-reversal step was previously reported. Recently, vesicle dynamics in large-amplitude oscillatory extensional flow was studied using a combination of precision flow experiments using the Stokes trap and numerical modeling and simulations. Overall, vesicle stretching in extensional flows generally involves distributed viscous forces across the entire membrane for freely suspended bodies in the absence of external forces. From this view, vesicle deformation in extensional flow is distinct from classical methods used to deform vesicle membranes, e.g., using optical tweezers to pull out thin lipid nanotubes through a localized point force on the membrane or microipette aspiration. Despite recent progress, the transient and steady-state dynamics of freely suspended vesicles in strong flows is not fully understood.

In this work, we directly observe the transient and steady-state stretching dynamics of freely suspended vesicles in extensional flow over a range of equilibrium morphologies including tubular, biconcave, and spheroidal shapes. Our results show that deformed vesicles adopt a steady-state elongated dumbbell shape in flow with highly deformed membrane conformations. We characterize vesicle stretching dynamics over a broad range of vesicle reduced volumes ($0.3 \leq \nu < 0.95$) by combining fluorescence microscopy with automated flow control, thereby enabling long time observation of vesicles with controlled strain-rate schedules. Using this approach, we observe a bevy of nonequilibrium steady-state morphologies for vesicles in strong flow, and we identify a second critical flow strength $\dot{\epsilon}_c$ above which vesicle stretch diverges in flow. The dependence of the critical flow strength $\dot{\epsilon}_c$ on vesicle reduced volume is characterized, and this behavior is directly compared to the coil-to-stretch transition for linear polymers and critical deformation of liquid drops in extensional flow. A simple analytical model is developed to describe vesicle membrane stretching dynamics in flow. Comparison with vesicle stretching data shows that the analytical model captures steady-state stretching of vesicles with larger volumes ($\nu \geq 0.7$); however, it fails to accurately describe the nonlinear stretching of highly deformed vesicles ($\nu < 0.7$). Implications for future development of analytical models are discussed in the context of highly deformed lipid bilayer vesicles.

MATERIALS AND METHODS

**Vesicle Preparation.** Giant unilamellar vesicles (GUVs) are prepared from a mixture of 1,2-dioleoyl-sn-glycero-3-phosphocholine (DOPC) and 0.12 mol % of the fluorescent lipid 1,2-dioleoyl-sn-glycero-3-phosphoethanolamine-N-(lissamine) rhodamine B sulfonyl (DOPE-Rh) in 100 mM sucrose buffer using an electroformation method, as previously described. Briefly, DOPC (25 mg/mL) is mixed with DOPE-Rh dye (0.04 mg/mL) and dissolved in chloroform to prepare a stock lipid solution. Next, 10 μL of this lipid mixture is deposited on a conductive indium tin oxide (ITO) coated glass slide (resistance $\approx 35 \Omega$, 25 × 50 × 1.1 mm, Delta Technologies) and dried under vacuum overnight. An electroformation chamber of volume $\approx 2.4$ mL was formed using the two ITO slides and a 1.5 mm Teflon spacer and was connected to a function generator (Hewlett-Packard/Agilent 33220 A). The electroformation chamber is filled with 100 mM sucrose solution (Sigma-Aldrich) and an alternating current (AC) electric field of 2 V/mm at 10 Hz is applied for 180 min at room temperature (22 °C). The majority of the vesicles prepared by electroformation method is nearly spherical in shape in the size range of 5–20 μm in radius. The viscosity of the 100 mM sucrose solution ($\lambda = 1.1$ mPa s) is measured using a benchtop viscometer (Brookfield) at 22 °C. To generate deformed vesicles, we used an osmotic deflation method which generates reduced volume vesicles in the range $0.30 < \nu < 0.90$, though the fraction of low reduced volume vesicles ($0.30 < \nu < 0.50$) in the suspension was relatively low. For experiments involving high solvent viscosities (viscosity ratio $\lambda = 0.1$), the viscosity of the suspending medium was increased to $\mu_{out} = 10.4$ mPa s by mixing the 100 mM sucrose solution with glycerol.

**Stokes Trap and Automated Flow Control.** A Stokes trap was used in conjunction with an inverted fluorescence microscope (Olympus IX71) on a vibration damping optical table (Thilors) (Figure 1). The microscopy setup included a 100 W mercury-arc lamp (USH1002D, Ushio), a 10X magnification air-immersion 0.45 NA objective lens, and a charge-coupled device (CCD) camera (PointGrey GS3-U3-120S6M-C). Standard techniques in soft-lithography are used to fabricate the microfluidic devices, as previously described. Microfluidic devices are mounted on the stage of an inverted microscope, and each of the four inlets of the flow device are connected to a fluidic reservoir (Elvelow XS) containing the buffer solution and the vesicle sample through perfluoroalkoxy (PFA) tubing with 1/16 in. outer diameter and 0.02 in. inner diameter (Figure 1A). Each output line from the fluidic reservoir is also connected to a high precision electronic pressure regulator (Elvelow OB1MKIII), which in turn is connected to a nitrogen cylinder with an output pressure of 30 psi for inducing a pressure driven flow in the microfluidic device.

Single vesicles are trapped in flow near the stagnation point of the microfluidic device using a feedback controlled algorithm (Figure 1B). In brief, freely suspended vesicles are confined in a planar extensional flow using the following approach: (1) images of vesicles in the microfluidic cross-slot device are captured using a CCD camera and vesicle center-of-mass positions are determined using image processing in real-time; (2) a target vesicle is confined in the two-dimensional (2D) flow plane by applying optimal flows through the four channels of the microfluidic device using a model-predictive control (MPC) algorithm. This enables precise confinement of vesicles under...
zero flow-rate conditions (using minor flows to counteract Brownian diffusion) or confinement under nonzero net flow conditions. Overall, the Stokes trap allows for the direct observation of vesicle stretching dynamics in precisely controlled flows (e.g., with precise control over the strain rate \( \dot{\epsilon} \)). To ensure that the flow field is well-characterized during vesicle stretching experiments, we determined the strain rate \( \dot{\epsilon} \) using particle tracking velocimetry and bead tracking methods, as previously described.29

**Results and Discussion**

**Direct Observation of Transient Stretching Dynamics in Flow.** We began by studying the stretching dynamics of nearly spherical vesicles \((\nu > 0.9)\) following the onset of extensional flow. Here, single vesicles are first trapped near the vicinity of the stagnation point in planar extensional flow in a cross-slot microfluidic device using a Stokes trap (Figure 1B). Vesicles are then observed under zero flow conditions for \(10–20 \text{ s} \) to determine reduced volume, as previously described.29,30 At time \( t = 0^\circ \), the vesicle is subjected to a sudden flow onset using a step strain-rate increase. In this way, vesicles are confined in a well-defined extensional flow for long periods of time \( t \) or accumulated fluid strain \( \epsilon = \int \dot{\epsilon} \text{d}t \) while directly observing transient stretching dynamics.30 Vesicle deformation is quantified by the aspect ratio \( A_i = L/L_0 = 1 \), where \( L_0 \) is the vesicle length at equilibrium. After the onset of flow, quasi-spherical vesicles transition to a steady ellipsoidal shape (Figure S1), consistent with prior work.13,29 In particular, the vesicle aspect ratio \( A_i \) initially increases linearly upon increasing Ca due to unfolding of membrane undulations, and then gradually approaches a maximum value (Figure S2). In all cases, vesicle relax back to the original equilibrium shape after the flow is switched off.

We next studied the transient stretching and relaxation dynamics of vesicles with tubular, biconcave, and spheroidal equilibrium shapes \((0.3 < \nu < 0.75)\) (Figure 2A). A characteristic series of images highlighting the conformational stretching dynamics of a single vesicle in flow as a function of time \((\nu = 0.64, \text{Ca} = 23)\) is shown in Figure 2A and Movie S1. Remarkably, vesicles are robustly deformed up to \(\approx 10–15\times\) their equilibrium size in flow. For \( \text{Ca} \) larger than a critical value \( \text{Ca}_{\text{cr}} \), viscous forces overcome bending stresses and drive a morphological transition to a dumbbell shape with a long, thin lipid nanotube connecting the two spherical ends. Interestingly, the radii of two spherical bulks remain nearly constant during the transient stretching process (Figure 2A). The increase in length of the thin lipid nanotube connecting the end bulbs results from local unfolding of membrane area from the spherical ends of the deformed vesicle. In this way, a shape change which takes the two ends connected by thin tether from a less symmetric to a more symmetric volume distribution contributes to the increase in nanotube length. Transient stretching trajectories are not greatly influenced by membrane thermal fluctuations because the bending modulus is significantly larger than thermal energy \((k_B T/\kappa < 0.04))\),29,30 and vesicles are exposed to strain rates \((\text{Ca} > \text{Ca}_{\text{cr}})\) much larger than the critical value required for dumbbell formation.

A series of transient stretching trajectories for vesicles with different reduced volumes is shown in Figure 2B. These results include the stretching dynamics of a single vesicle \((\nu = 0.71)\) subjected to repeated stretch-relax deformation cycles \((\text{Ca} = 39.7, 79.4, 119.1)\), which shows that the rate of membrane stretching increases with increasing \( \text{Ca} \) in the first cycle \((\text{Ca} = 39.7)\).
39.7), the vesicle deforms and approaches a steady-state aspect ratio \( A_r \approx 5 \) after 50−60 s. In the second and third cycles (\( Ca = 79.4 \) and 119.1), the vesicle rapidly deforms to \( A_r \approx 10 \) in only a few seconds, after which the vesicle stretches beyond the field-of-view of the microscope, which generally limits observation of steady-state vesicle deformation to <0.5 mm. Strikingly, these results show that deflated vesicles are highly deformable in flow, such that vesicle membrane integrity is maintained upon repeated deformation in strong flows. In general, this behavior is also observed at a viscosity ratio of \( \lambda = 0.1 \) (Figure S3).

**Steady-State Stretching in Flow and Thin Lipid Nanotubes.** Next, we characterized the steady-state deformation of vesicles in flow (Figure 3, Figure S4). Using the Stokes trap, single vesicles are subjected to a series of step increases in strain rate \( \dot{\varepsilon} \), thereby enabling determination of the steady-state extension for a single vesicle with reduced volume \( \nu \) as a function of \( Ca \). At each step increment of \( \dot{\varepsilon} \), the strain rate is held constant for at least 10 s to characterize the steady-state deformation of the vesicle in flow. A characteristic multistep strain-rate experiment for a tubular vesicle (\( \nu = 0.54 \)) is shown in Figure 3A,B. Here, the tubular vesicle is initially trapped at \( Ca = 0 \) and \( Ca = 2.8 < Ca_{c1} \), which is smaller than the critical \( Ca \) required to induce a transition to a dumbbell conformation \(^{29}\) (Figure 3A). The flow rate is then increased in a stepwise fashion to \( Ca = 5.6 \), after which the vesicle undergoes a morphological change to a symmetric dumbbell with a thin lipid tether connecting two spherical bulbs on the ends of the stretched vesicle. Further stepwise increases of the flow strength to \( Ca = 8.4 \) and 11.2 lead to additional deformation of the vesicle in flow (Figure 3A,B). In these experiments, the steady-state value of the aspect ratio \( A_r \) is determined as the average of vesicle stretch in the plateau region of the trajectory after transient stretching has ended, with the error bar determined as the standard deviation around the mean.

In prior work, boundary integral simulations were used to study vesicle conformations in extensional flow\(^ {27}\) and accurately captured the dynamics in the vicinity of \( Ca \approx Ca_{c1} \), where vesicle shape transitions from the equilibrium conformation to a dumbbell shape. However, prior numerical simulations and experiments reported no clear steady-state deformation for anisotropic vesicles in strong flow (\( Ca > Ca_{c1} \)). In contrast, our

![Figure 2.](image1)

**Figure 2.** Transient stretching and relaxation dynamics of single vesicles in flow. (a) Snapshots showing temporal evolution of vesicle shape (\( \nu = 0.64 \)) in extensional flow at \( Ca = 23 \) showing the formation of an asymmetric dumbbell with a long nanotube tether connecting two spherical end bulbs. The scale bar is 30 \( \mu \)m (SI Movie 1). (b) Transient stretching and relaxation (\( L/L_0 - 1 \)) trajectories of vesicles as a function of time \( t \) for a wide range of reduced volume \( \nu \) and capillary number \( Ca \). Here, the viscosity contrast is \( \lambda = 1 \).

![Figure 3.](image2)

**Figure 3.** Characterization of steady-state dynamics for different vesicle morphologies. (a) Snapshots of a tubular vesicle (reduced volume \( \nu = 0.54 \pm 0.02 \)) showing steady-state dumbbell shapes as a function of \( Ca \). The scale bar is 20 \( \mu \)m. (b) Transient deformation of the tubular vesicle in part a as a function of time upon a stepwise increase in \( Ca \). (inset) Strain rate as a function of time. (c) Steady-state aspect ratio \( A_r \) of vesicles as a function of \( Ca \) for several reduced volumes \( \nu \) at a viscosity ratio \( \lambda = 1 \). The aspect ratio for vesicles with \( \nu < 0.75 \) increases rapidly in a narrow range of \( Ca \) beyond a critical value \( Ca_{c1} \), required to induce a symmetric or asymmetric dumbbell. Error bars represent ±1 standard deviation from the mean value.
Error bars represent large nonlinear deformation in a narrow range of arises due to large availability of excess membrane area. With high deformability of low reduced volume vesicles, which multiple steady-state conformations upon increasing morphological transition to a dumbbell shape, vesicles exhibit de

Above a second critical capillary number \( \text{Ca}_{\text{crit}} \), polymers, and liquid drops as a function of normalized flow strength \( \text{Ca}/\text{Ca}_{\text{crit}} \) for vesicles, \( \text{Ca}/\text{Ca}_{\text{crit}} \) for drops, \( \text{Wi}/\text{Wi}_{\text{crit}} \) for polymers. Error bars represent \( \pm 1 \) standard deviation from the mean value. (b) Log–log plot of \( \text{Ca}_{\text{crit}} \) as a function of reduced volume \( \nu \) for viscosity ratio \( \lambda = 1 \). Red squares represent the experiment, and the black line is a power-law fit of the form \( \text{Ca}_{\text{crit}} = a \nu^b \). (inset) Critical capillary number as a function of drop radii for liquid drops suspended in Newtonian and non-Newtonian fluids. 

Figure 4. Characterization of critical capillary number \( \text{Ca}_{\text{crit}} \). (a) Normalized aspect ratio \( A_r \), fractional extension, and deformation parameter \( D \) for vesicles (\( \lambda = 1 \)), polymers, and liquid drops as a function of normalized flow strength \( \text{Ca}/\text{Ca}_{\text{crit}} \) for vesicles, \( \text{Ca}/\text{Ca}_{\text{crit}} \) for drops, \( \text{Wi}/\text{Wi}_{\text{crit}} \) for polymers. Error bars represent \( \pm 1 \) standard deviation from the mean value. (b) Log–log plot of \( \text{Ca}_{\text{crit}} \) as a function of reduced volume \( \nu \) for viscosity ratio \( \lambda = 1 \). Red squares represent the experiment, and the black line is a power-law fit of the form \( \text{Ca}_{\text{crit}} = a \nu^b \). (inset) Critical capillary number as a function of drop radii for liquid drops suspended in Newtonian and non-Newtonian fluids. 

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Experiments show that vesicles exhibit highly deformed steady-state morphologies over a range of reduced volumes. Broadly speaking, these experiments are uniquely enabled by automated flow control via the Stokes trap, which allows for direct observation of deformed, freely suspended vesicles in well-defined flows for long times.

We systematically investigated the steady-state aspect ratio \( A_r \) of moderately deformed (0.6 < \( \nu < 0.75 \)) and highly deformed (\( \nu < 0.6 \)) vesicles over a wide range of \( \text{Ca} \) at \( \lambda = 1 \) (Figure 3C). Vesicles with reduced volume \( \nu < 0.75 \) do not appreciably deform below a critical \( \text{Ca} < \text{Ca}_{\text{crit}} \) as highlighted by vertical dashed line in Figure 3C, which is consistent with prior work. Above a critical value of \( \text{Ca} > \text{Ca}_{\text{crit}} \), required for the morphological transition to a dumbbell shape, vesicles exhibit multiple steady-state conformations upon increasing \( \text{Ca} \). The large nonlinear deformation in a narrow range of \( \text{Ca} \) is consistent with high deformability of low reduced volume vesicles, which arises due to large availability of excess membrane area. Above a second critical capillary number \( \text{Ca}_{\text{crit}} \), the aspect ratio \( A_r \) diverges, highlighted by vertical black dashed line for \( \nu = 0.74 \) vesicle in Figure 3C. At even higher values of \( \text{Ca} \) than shown in Figure 3, vesicles presumably adopt additional highly deformed states, though our observations are limited by the finite field-of-view of the microscope. Transient stretching experiments on vesicles with viscosity contrast \( \lambda = 0.1 \) show a similar overall trend of steady-state deformation as a function of \( \text{Ca} \) for vesicles with varying reduced volumes (Figure S5).

We further compare our observations for nonlinear vesicle stretching in flow to other soft materials such as single polymer molecules. In extensional flow, linear polymers are known to undergo a conformational transition from the coiled to stretched state at a critical dimensionless flow strength known as the Weissenberg number \( \text{Wi}_{\text{crit}} \approx 0.5 \), which roughly corresponds to \( \approx 25\% \) of the \( \text{Wi} \) at which the polymer stretch reaches a plateau around \( \text{Wi} \approx 2 \). Similarly, in our work, the observed values of \( \text{Ca}_{\text{crit}} \) correspond to at least 25\% of the critical capillary number \( \text{Ca}_{\text{crit}} \), at which vesicle stretch \( A_r \) is predicted to saturate due to the straightening out of all membrane folds in flow (Supporting Information). In particular, we estimate that the maximum length associated with unfolding of all membrane fluctuations lies between 3 mm and 10 mm dependent on vesicle size and reduced volume (Supporting Information). In this way, the observed values of \( \text{Ca}_{\text{crit}} \) in our experiments lie between 30 and 80\% of the predicted critical capillary number \( \text{Ca}_{\text{crit}} \), at which all of the membrane folds are pulled out in flow. Thus, we take \( \text{Ca}_{\text{crit}} \) as a critical capillary number for \( A_r \) divergence and the corresponding critical deformation as \( A_{r,crit} \).

Characterization of Critical Capillary Number \( \text{Ca}_{\text{crit}} \). We investigated the nature of the critical capillary number \( \text{Ca}_{\text{crit}} \) by directly comparing the behavior for vesicles with polymers and liquid drops in extensional flow (Figures 4A,B and S6). Figure 4A plots the normalized aspect ratio \( A_r/A_{r,crit} \) as a function of reduced capillary number \( \text{Ca}/\text{Ca}_{\text{crit}} \) for vesicles with different \( \nu \). Interestingly, our results show that the transition to critical deformation \( A_r \to A_{r,crit} \) sharpens as the reduced volume is decreased in the limit of \( \text{Ca} \to \text{Ca}_{\text{crit}} \). This is consistent with the notion that vesicles with small reduced volumes have large excess area and can undergo abrupt deformations in flow. Clearly, a vesicle’s reduced volume \( \nu \) and equilibrium morphology affect the critical stretching dynamics in flow, thereby influencing the nonlinear stretching and critical capillary number \( \text{Ca}_{\text{crit}} \).

To directly compare vesicle dynamics with other materials in flow, we plot the normalized deformation parameter \( D/D_{crit} \) for liquid drops and fractional extension for linear polymers in steady extensional flow (Figure 4A). In flow, liquid drops...
suspended in an immiscible Newtonian fluid are known to exhibit a steady-state deformation (D) up to a critical capillary number, after which drop shape diverges. As the capillary number approaches the critical value (Ca \rightarrow Ca_{crit}) the sharpness of transition to critical deformation D \rightarrow D_{crit} for liquid drops in a Newtonian fluid exhibits a universal plot independent of drop size, shown by the green circles in Figure 4A. This universality is lost for drops suspended in non-Newtonian fluids, as the transition to D_{crit} becomes sharper for liquid droplets with smaller radii due to increasing viscoelastic effects (green squares and diamonds in Figure 4A). Remarkably, these results reveal that the abruptness of transition A \rightarrow A_{crit} for vesicles is enhanced for smaller reduced volumes due to increasing membrane flappiness; however, the sharpness of transition D \rightarrow D_{crit} for liquid drops is enhanced for smaller radii due to increasing viscoelastic effects in non-Newtonian fluids.

We also compared the critical stretching dynamics of vesicles to linear polymers in extensional flow, where the polymer contour length L and polymer solution concentration c are known to impact the sharpness of the coil-to-stretch transition (CST). As shown in Figure 4A, high molecular weight genomic DNA (L \approx 1.3 \text{ mm}) exhibits a sharper CST as Wi \rightarrow Wi_{crit} compared to A-DNA (L \approx 21.1 \text{ \mu m}) in ultradilute solutions (c \approx 10^{-5} \text{ c}^6). In addition, increasing the background polymer concentration from 10^{-5} \text{ c}^6 to \approx 1 \text{ c}^6 tends to inhibit polymer stretching, such that the CST is far less sharp than ultra dilute solutions as Wi \rightarrow Wi_{crit}. From this view, the role of decreasing vesicle reduced volume \nu on the steady-state deformation A_{crit} of vesicles (i.e., increased membrane fluctuations and deformability) is analogous to the role of increasing polymer contour length L on the CST transition in extensional flow (i.e., increased polymer extensibility).

We further quantified the critical capillary number Ca_{crit} as a function of vesicle reduced volume \nu (Figure 4B). Interestingly, our results show that Ca_{crit} follows a power law with \nu such that Ca_{crit} = a \nu^{1/2}, where a is a constant and \beta is the power-law exponent. Our results show that Ca_{crit} for vesicles follows a power-law dependence on reduced volume, which differs from the behavior of liquid drops, where the critical capillary number is independent of drop size in Newtonian fluids and varies linearly with drop-size in non-Newtonian liquids.

**Model for Steady-State Vesicle Deformation.** We further sought to develop an analytical model to understand vesicle deformation and membrane elastic properties. For deformed vesicles with symmetric or asymmetric dumbbell shapes, the free energy of a thin nanotube connecting the two spherical bulbs at steady-state (Figure S7) can be written as

\[ E = \frac{1}{2} \frac{\nu}{r^4} 2 \pi \nu r^4 L + 2 \pi \nu r^4 L_{0} - f L_{0} \]

where \( r_{i} \) is the nanotube radius, \( L_{i} \) is the nanotube length, \( f \) is the hydrodynamic stretching force, and \( \nu \) is the membrane tension. Under steady-state conditions, the tube radius and stretching force are estimated by taking the derivative of the free energy such that \( \frac{\partial f}{\partial r} = 0 \) and \( \frac{\partial f}{\partial L} = 0 \). In this way, the steady-state tether radius \( r_{i} \) and force \( f \) are expressed as \( r_{i} = \sqrt{\nu/2f} \) and \( f = 2\pi \nu/2\sigma \). Moreover, the increase in tension \( \sigma \) under viscous loading occurs due to an increase in the vesicle area, which can be expressed as

\[ \sigma = \sigma_{0} \exp \left( \frac{8\pi \nu A - A_{0}}{k_{B}T A_{0}} \right) \]

where \( \sigma_{0} \) is the initial membrane tension, \( k_{B} \) is the Boltzmann constant, \( T \) is the absolute temperature, \( A \) is the observed membrane area at steady-state, and \( A_{0} \) is the area of initial vesicle deformation (Figures S7 and S8). For a given strain rate \( \dot{\epsilon} \), the maximum membrane tension \( \sigma_{max} \) is achieved when the viscous forces on the spherical bulb are balanced by the tether stretching force \( f \). Thus, the force balance is written as \( \sigma_{max} = 2\pi \nu/2\sigma_{max} \), where \( R_{i} \) is the radius of upper spherical end (Figure S7). Therefore, the nanotube radius corresponding to the maximum tension is given as \( r_{i,\min} = \nu/3\mu R_{i}^{2} \). As previously discussed, the radii of two spherical end bulbs (\( R_{i} \) and \( R_{e} \)) remain approximately constant during the stretching experiments (Movie S1). Moreover, we assume a simplified vesicle geometry immediately at the onset of conformation changes in flow, as shown in Figures S7 and S8. In this way, the membrane area \( A \) at steady-state is given as \( A = 4\pi R_{i}^{2} + 4\pi R_{e}^{2} + 2\pi r_{i}L_{0} \) and the initial membrane area is \( A_{0} = 4\pi R_{i}^{2} + 4\pi R_{e}^{2} \). Substituting these results into eq 2, the steady-state nanotube
length $L$ is given as a function of the strain rate $\dot{e}$ and vesicle parameters:

$$
L = \frac{3\mu k_T}{4\pi^2} \ln \left( \frac{3\mu k_T}{2\sigma_k} \right)
$$

In eq 3, $\kappa$ and $\sigma_k$ are membrane properties namely the bending modulus and membrane tension, while the remaining variables are constants related to vesicle geometry and solvent viscosity.

**Model Prediction of Steady-State Vesicle Deformation in Flow.** We directly compare experimental data for nonlinear vesicle stretching at steady-state to the analytical model given in eq 3 (Figure 5). Here, the steady-state nanotube lengths of several vesicles with different reduced volumes ($\nu = 0.73, 0.67, 0.54$) are plotted as a function of strain rate $\dot{e}$ with the corresponding analytical model predictions from eq 3. Using fluctuation spectroscopy, we determined the ensemble-averaged bending modulus $\kappa$ for quasi-spherical vesicles to be $\kappa = 22.3 k_B T$ and the average surface tension to be $\sigma = 10^{-7}$ N/m, as described in prior work.\(^2\) In this work, the deformed vesicles are nonspherical at equilibrium, but it is common to assume that membrane properties such as $\kappa$ for nearly spherical vesicles are relatively constant regardless of global vesicle shape.\(^2\) Overall, the experimental steady-state stretch for vesicles with $\nu = 0.73$ is captured reasonably well by the analytical model (eq 3) (Figure 5A), presumably because the vesicle is moderately deformed and closer to a nearly spherical shape ($\nu \approx 1$).

We further aimed to understand if the experimental steady-state stretching data could be accurately captured by the analytical model for vesicles with smaller reduced volumes $\nu$ (e.g., for highly deformed vesicles). However, our results show that the analytical model fails to adequately describe the nonlinear steady stretching dynamics for vesicles with smaller reduced volumes (Figure 5B,C). In particular, the steady-state stretching data for highly deformed vesicles ($\nu = 0.54$) is not well described by the analytical model assuming a constant value of the bending modulus $\kappa$. Prior work has shown that individual vesicles with the same membrane composition can exhibit wide variability in bending modulus $\kappa$, such that vesicles prepared by identical procedures in different experimental trials can show $\kappa$ values that vary by a factor of 2.\(^2\) We therefore evaluated the analytical model for vesicles with $\nu = 0.54$ using a smaller value of $\kappa = 11 k_B T$. However, the analytical model was unable to accurately capture the experimental data, severely underpredicting the steady-state nonlinear stretch of vesicles with small reduced volumes. Overall, these observations suggest a few key limitations in the analytical model. First, the analytical model ignores thermal fluctuations of the membrane; however, vesicles with smaller reduced volumes show enhanced thermal fluctuations (SI Movie 2), which suggests that membrane fluctuations should be considered in the development of more accurate physical models of highly nonlinear membrane stretching for deformed vesicles. In addition, the model does not include contributions from the spontaneous curvature of the membrane. Prior work has shown that such effects may be important for highly deformed vesicles considered in our experiments.\(^1,3,8,6,7\) Moreover, area-difference elasticity\(^7,8\) may be important for large tube lengths (>300 $\mu$m) considered in our experiments. Overall, our results suggest that deformed vesicles with $\nu < 0.7$ undergo extreme nonlinear deformation within a relatively small of strain rates or $\nu$. In developing new realistic models for nonlinear membrane stretching, future work should consider membrane undulations, spontaneous curvature, and area-difference elasticity to accurately capture the nonlinear dynamics of deformed vesicles.

**CONCLUSIONS**

In this work, fluorescence microscopy combined with automated flow control was used to precisely characterize the transient and steady-state nonlinear stretching dynamics of vesicles in flow. Automated flow control offers several advantages for understanding vesicle dynamics in nonequilibrium flows. In particular, the Stokes trap allows for the long time observation of freely suspended vesicles without the need for micropipettes or surface attachment to the membrane interface. In addition, precise control over the flow rate allows for systematic examination of highly deformed steady-state conformations of vesicles over a broad range of reduced volumes (0.3 < $\nu$ < 0.95) and capillary numbers $\nu$. In particular, our work has clearly revealed the nonlinear stretching dynamics in strong flows such that $\nu \gg \nu_c$.

Using this approach, we characterize the steady-state deformation of vesicles in flow. Our results show that vesicles with reduced volume $\nu < 0.75$ adopt highly elongated dumbbell shapes in strong flows above a critical capillary number $\nu_c$, wherein the vesicle steady-state stretch increases nonlinearly with capillary number in flow. Interestingly, our results show that anisotropic vesicles are highly deformable objects, stretching to several millimeters in length in flow, which corresponds to 100–120 times their equilibrium size. The steady-state dynamics of deformed vesicles ($\nu < 0.75$) in flow starkly contrasts with the dynamics of nearly spherical vesicles ($\nu > 0.8$), where the deformed aspect ratio initially increases linearly with capillary number before saturating. Moreover, nearly spherical vesicles adopt a deformed ellipsoid shape in flow, whereas all deformed vesicles adopt a dumbbell shape consisting of two spherical end bulbs connected by a thin tether.

Remarkably, our results reveal a second critical capillary number $\nu_c$, above which the vesicle aspect ratio diverges for a wide range of reduced volumes ($\nu < 0.75$). Our results show that $\nu_c$ follows a power-law dependence on reduced volume, which fundamentally differs from the steady-state dynamics of liquid drops in non-Newtonian fluids, where the critical capillary number varies linearly with drop radius, or liquid drops in Newtonian fluids, where the critical capillary number is independent of drop radius. For liquid drops in non-Newtonian fluids, the transition to the critical deformation parameter $D \rightarrow D_{c,\nu}$ becomes less sharp down in the limit $\nu \rightarrow \nu_{crit}$ with increasing drop radius due to a reduction in viscoelastic effects of the suspending fluid. Similarly, in the limit of $\nu \rightarrow \nu_{crit}$, the sharpness of transition to critical behavior $A \rightarrow A_{crit}$ decreases upon increasing the reduced volume. Overall, these results suggest that the transition to $A_{crit}$ for vesicles with large reduced volumes becomes less abrupt due to a reduction in membrane deformability. Interestingly, polymers also show a less sharp coil-to-stretch transition in extensional flow when the polymer chain extensibility is decreased, e.g. for lower molecular weight polymers. Thus, in the context of critical deformation dynamics in flow, the role of increased reduced volume for vesicles is analogous to the role of decreased molecular weight for polymers and increased radii for liquid drops in non-Newtonian solvents.

Our results provide a direct link between membrane mechanics and vesicle deflation. An analytical model based on
only two membrane properties (bending modulus and surface tension) is developed to understand the experimental data, which allows for comparison of the steady-state stretching data to theory for varying amounts of vesicle deflation. Our results show that the analytical model fails to accurately capture the nonlinear vesicle dynamics upon decreasing the reduced volume. These results suggest that additional membrane properties and phenomena such as spontaneous curvature, Gaussian bending modulus, and thermal fluctuations may be important for highly deflated vesicles. Overall, these results provide new insights into the vesicle nonlinear dynamics in strong flows, which is an important step forward in understanding membrane properties for vesicles with different equilibrium morphologies.

■ ASSOCIATED CONTENT

* Supporting Information

The Supporting Information is available free of charge at https://pubs.acs.org/doi/10.1021/acs.langmuir.1c01275.

Supplementary figures of the transient dynamics of nearly spherical vesicles (PDF)

Supplementary movies as mentioned in the text (ZIP)

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Notes

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