Nonmonotonic dependence of comb polymer relaxation on branch density in semidilute solutions of linear polymers

Shivani F. Patel,1,* Charles D. Young,1,* Charles E. Sing,1 and Charles M. Schroeder1,2,†

1Department of Chemical and Biomolecular Engineering and Beckman Institute for Advanced Science and Technology, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, USA
2Department of Materials Science and Engineering, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, USA

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The dynamics of branched polymers in semidilute solutions are thought to depend on a coupling between polymer chain architecture, intermolecular excluded volume interactions, and long-range hydrodynamic interactions (HI). However, it has been challenging to quantitatively understand these phenomena due to the influence of intra- and intermolecular HI and chain-chain interactions for nonlinear polymer topologies in flow. In this work, we directly observe the relaxation dynamics of comb-shaped polymers in semidilute solutions of linear polymers using single molecule experiments and molecular simulations. Experimental results are directly complemented by coarse-grained simulations of comb polymers in semidilute solutions of linear polymers including long-range HI and excluded volume interactions. Our results show an unexpected nonmonotonic dependence of comb polymer relaxation time on branch density that arises due to a subtle yet important interplay between hydrodynamic shielding and polymer architecture.

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Branched polymers exhibit complex flow behavior due to nonlinear chain architectures [1]. The dynamics of branched polymers are qualitatively different than linear chains due to the presence of branch points [2]. In concentrated solutions and melts, linear polymers are known to topologically entangle and relax stress by reptation, which involves the snakelike motion of chains along the backbone contour [3]. However, branch points significantly slow down the relaxation of comb-shaped polymers [4], giving rise to hierarchical relaxation mechanisms that differ from those in linear polymer melts [5]. Such behavior gives rise to distinct rheological responses such as enhanced strain hardening together with shear thinning [6,7], unique linear viscoelastic signatures [8,9], and double stress overshoots in the startup of shear [10,11].

Branched polymer solutions are extensively used in applications such as consumer care products [12,13]. Moreover, nondilute solutions of branched polymers have been used for chemical sensing [14], optically active materials [15], and in emerging technologies such as three-dimensional printing [16]. Despite the utility of branched polymers, we lack a complete understanding of their dynamics in semidilute unentangled solutions, wherein polymer coils interpenetrate but do not topologically entangle [3]. In dilute polymer solutions, bulk properties arise from single isolated chain behavior, whereas in highly entangled concentrated solutions and melts, hydrodynamic interactions (HI) are screened and interchain interactions can be described using a mean-field approach [3]. In
FIG. 1. Single molecule experiments and Brownian dynamics (BD) simulations of comb polymer relaxation in semidilute solutions. (a) Schematic of experimental system (top) showing dual-labeled comb polymers with backbones ($L_{bb} = 17.4 \, \mu m$) and branches ($L_{br} = 0.9 \, \mu m$) and (bottom) molecular model for BD simulations. (b) Schematic of relaxation experiment. (c) Top: Single molecule snapshots of a comb polymer with $n_{br} = 5$ branches relaxing after flow cessation. The red laser is turned off for $t \geq 4$ s to minimize photobleaching. Bottom: Snapshots from BD simulations showing comb relaxation ($n_{br} = 4$, green/red) in a background of semidilute linear polymers (gray) at equivalent time intervals as the experiment.

In recent years, single molecule fluorescence microscopy has been used to directly observe polymer dynamics at the level of individual chains [17], enabling systematic studies in well-defined flows [18,19]. Concurrently, Brownian dynamics (BD) simulations have been used to probe the molecular origins of polymer flow behavior [20–22]. Single molecule experiments and BD simulations have yielded new insights into the dynamics of architecturally complex polymers such as combs [23,24], knots [25,26], and rings [27,28], mainly in dilute solutions. These approaches have also been extended to semidilute solutions, though prior work has focused on linear polymers [19,29–31].

In this work, we use single molecule fluorescence microscopy and BD simulations to understand the role of branching on polymer relaxation in semidilute solutions. Here, we directly visualize the relaxation dynamics of fluorescently labeled DNA comb polymers as a function of branching density in semidilute solutions of linear chains. Our results show an unexpected nonmonotonic dependence of comb polymer relaxation time on branch density, which is modeled using BD simulations of semidilute solutions with and without HI. BD simulations are performed across a wide range of branch density, branch lengths, and polymer concentrations, providing key insight into the molecular origins of the unexpected relaxation behavior of comb polymers. Taken together, our results show that comb polymer relaxation in semidilute solutions is governed by a complex interplay between long-range HI and polymer architecture.

DNA-based comb polymers were synthesized using a hybrid enzymatic-synthetic approach, as previously described (Fig. 1) [23,32,33]. Briefly, polymerase chain reaction (PCR) is used to generate the comb polymer backbones and branches with desired monodisperse lengths in separate reactions. Backbones ($M_{bb} = 40 \, kbp$) are synthesized with non-natural nucleotides enabling a graft-onto reaction for side branches via “click” (azide-alkyne) reaction. Branches ($M_{br} = 2.2 \, kbp$) are
FIG. 2. Relaxation of comb polymers in semidilute solutions of linear chains. Ensemble-averaged relaxation trajectories of comb polymers from experiments \((M_{bb} = 40 \text{ kbp}, M_{br} = 2 \text{ kbp})\) as a function of branch number \(n_{br}\) compared to their linear analogs \((M_{bb} = 40 \text{ kbp})\).

The comb polymers are synthesized by incorporating Cy5 dye (red emission) during PCR, together with a single chemical group (azide) at one terminus enabling the graft-onto reaction. Following synthesis and purification, comb polymers are labeled with YOYO-1 dye (green emission), thereby allowing for simultaneous two-color imaging [23].

Polymer solutions are prepared by adding trace amounts of combs to semidilute unentangled solutions of linear polymers [Fig. 1(a)]. The unlabeled background solution consists of monodisperse linear DNA (\(\lambda\)-phage DNA, \(M_{bb,\lambda} = 48.5 \text{ kbp}\)) at a concentration \(c = 1 \text{ c}^* = 50 \text{ ng/\mu l}\), where \(c^*\) is the overlap concentration that defines the crossover from the dilute to the semidilute regime [30]. Comb polymers are added at an extremely low concentration corresponding to \(c \approx 10^{-5}c^*\) relative to the linear polymer background. The contour lengths of fluorescently labeled comb backbones and branches are \(L_{bb} = 17.4 \mu m\) and \(L_{br} = 0.9 \mu m\), respectively, with a persistence length \(l_p = 56 \text{ nm}\) [17]. The contour lengths of the labeled comb backbone \(L_{bb}\) and unlabeled linear chains \((L = 16.3 \mu m, \lambda\)-phage DNA) are nearly equivalent [17].

We observed the relaxation of comb polymers following a step strain deformation in extensional flow [Figs. 1(b) and 1(c)]. During stretching, single comb polymers are trapped near the stagnation point in planar extensional flow using an automated flow control device known as the Stokes trap [34–38]. In this way, comb polymers are stretched to high degrees of extension \((\approx 0.6–0.7L_{bb})\) for a time duration \(t\) using strain rates \(\dot{\epsilon}\) above the coil-stretch transition for large fluid strains \(\epsilon = \dot{\epsilon}t \approx 12–15\). Prior to flow cessation, the number of branches on individual comb polymers is directly counted. Using this approach, we characterized the relaxation of comb polymers \((n_{br} \geq 1 \text{ branches})\) and linear polymers \((n_{br} = 0 \text{ branches})\) as a function of branching density (Fig. 2 and Supplemental Material Fig. S1 [32]). Here, we constructed separate molecular ensembles for different polymer architectures, including molecular ensembles for the cases of \(n_{br} = 0, 1, 2, 3\) branches, and a combined molecular ensemble for \(n_{br} = 4–8\). Ensemble-averaged trajectories from experiments for different
FIG. 3. Molecular ensembles of comb polymer relaxation as a function of the number of branches $n_{br}$ from experiments. (a)–(f) Relaxation dynamics of DNA comb polymers in semidilute 1$c^*$ solutions of linear chains following cessation of extensional flow. Relaxation dynamics are shown by plotting the fractional extension as a function of time for each branching density ($n_{br} = 0, 1, 2, 3, 4, 5–8$). Individual relaxation trajectories are shown in gray and the ensemble-averaged trajectories are shown in blue.

Relaxation trajectories for linear polymers are well described by a single exponential decay for $x/L < 0.3$, such that $(x/L)^2 = A \exp(-t/\tau) + B$, where $\tau$ is the longest relaxation time and $A$ and $B$ are fitting constants [30]. Our results show that the relaxation trajectories for comb polymers in semidilute solutions of linear chains are well described by a single exponential (Supplemental Material Fig. S2), as shown previously for combs in dilute solution (Supplemental Material Fig. S3) [23].

The ensemble-averaged relaxation time for linear chains ($n_{br} = 0$) is $\tau = 6.3 \pm 0.2$ s. The average relaxation times for comb polymers with low branching density ($n_{br} = 1, 2, 3$) were found to be similar with $\tau = 5.0 \pm 0.3$ s. As branch density increases ($n_{br} \geq 4$), comb relaxation time increases such that $\tau = 5.9 \pm 0.2$ s. Error bars are determined as the standard error of the mean from individual trajectories and represent statistical variations arising from stochastic single molecule dynamics.

Unexpectedly, our results for comb polymer relaxation (Fig. 2, inset) show that comb polymers with a small number of branches ($n_{br} = 1, 2, 3$) relax faster than linear polymers ($n_{br} = 0$) in semidilute solutions. However, increasing the number of branches on comb polymers ($n_{br} \geq 4$) slows down the relaxation dynamics. These results are statistically significant such that differences in the ensemble-averaged relaxation times are larger than the standard error in molecular ensembles (Fig. 2, inset). This nonmonotonic trend in comb polymer relaxation times as a function of branch density is shown in Fig. 2. Each ensemble contains at least 50 molecular trajectories, as shown in Fig. 3.
density is unexpected given that linear polymers relax faster than their comb counterparts in dilute solutions [23] and melts [7].

To understand the molecular origins of comb relaxation dynamics, we used BD simulations with HI. Here, polymers are modeled with $N_{bb} = 100$ backbone beads and $N_{br} = 6, 13,$ or 26 branch beads, connected by Kremer-Grest springs [32,40], and a varying number of branches $n_{br} = 0, 1, 4, 7$ [Fig. 1(a)]. In this model, the Kuhn step size is $l_K \approx 1.8 \sigma$, where $\sigma$ is the bead diameter. Comb backbones and branches in BD simulations have a proportionally smaller number of Kuhn segments compared to experiments, such that the ratio of backbone to branch length matches experiment ($M_{br}/M_{bb} \approx N_{br}/N_{bb}$). Intrachain and interchain HI is modeled using an Ewald sum Rotne-Prager-Yamakawa tensor [41,42]. BD simulations of nondilute solutions with HI are computationally demanding due to the calculation of HI and correlated Brownian noise; to accelerate the simulations, we use the iterative conformational averaging method [31,43,44], where the decomposition of the diffusion tensor is avoided by conformationally averaging the Brownian noise over transient polymer conformations [31,44]. In particular, the Brownian noise is determined by a conformationally averaged form of the truncated expansion ansatz introduced by Geyer and Winter [32,45]. Prior work has shown that such coarse-grained models successfully predict qualitative trends in experiments [27,31,44].

In an identical procedure to experiments, BD simulations are initialized with random polymer configurations and equilibrated for $10 \tau$. A planar extensional flow is then applied using Kraynik-Reinelt boundary conditions [46,47] for an accumulated strain $\epsilon = 20–25$ at a flow strength $Wi > 1.5$, where $Wi = \dot{\epsilon} \tau$ is the dimensionless flow strength known as the Weissenberg number. In this way, polymers are stretched to a fractional extension $x/L > 0.5$, followed by abrupt flow cessation and relaxation of polymers to their equilibrium state (Fig. 4). During the relaxation step, the backbone end-to-end distance is tracked over time, enabling determination of the longest polymer relaxation times $\tau_L$. Simulation ensembles contain 300 molecules, and error bars are determined from the standard error of four subsamples of the ensemble.

Figure 5 shows the longest relaxation times from experiments and BD simulations ($N_{br} = 6$). Here, relaxation times are nondimensionalized with the longest relaxation time of linear polymers ($\tau_L$) to directly compare simulation and experiment. Both experiments and BD simulations with HI show the overall nonmonotonic dependence of polymer relaxation time on branch density. Compared to experimental data, BD simulations with $N_{br} = 6$ exhibit a smaller decrease in relaxation time between linear chains ($n_{br} = 0$) and combs with one branch ($n_{br} = 1$); we attribute this disparity to model coarse graining, because the nonmonotonicity becomes more pronounced with increasing branch length [$N_{br} = 6, 13,$ to 26; Fig. 5(b)]. Overall, BD simulations reveal that both interpolymer interactions and HI give rise to the nonmonotonic relaxation time. Increas-
Comb polymer relaxation time as a function of the number of branches $n_{br}$ in semidilute solutions of linear polymers. (a) Normalized average relaxation time ($\tau_c/\tau_L$) of comb polymers from experiments (2.2 kbp branches) and simulations ($N_{br} = 6$). Molecular ensembles for comb polymers with $n_{br} \geq 4$ branches are combined and shown as $n_{br} = 5^*$. (b) Effect of branch size; scaled relaxation time for comb polymers in semidilute solutions from BD simulations with HI for branch sizes $N_{br} = 6, 13, \text{and } 26$. (c) Effect of concentration; scaled relaxation time for comb polymers from BD simulations with HI in ultradilute and semidilute solutions at 0.1, 1.0$c^*$. (d) Effect of HI; scaled relaxation time for comb polymers in semidilute solutions from BD simulations with and without HI for branch size $N_{br} = 13$.

Based on these results, we attribute the unusual relaxation behavior of comb polymers in semidilute solutions to an interplay between hydrodynamic screening and intermolecular excluded volume in nondilute polymer solutions. We hypothesize that comb polymer branches may induce a local dilution in the vicinity of their backbones due to steric interactions with nearby polymer chains in the background solution. To test this hypothesis, we performed BD simulations with and without HI for linear ($n_{br} = 0, N_{bb} = 100$) and comb polymers ($n_{br} = 1, N_{br} = 6, 13, \text{and } 26, N_{bb} = 100$). We determined the equilibrium intermolecular radial distribution function $g(r)$ for a monomer located at a branch point for a comb with a single branch ($n_{br} = 1$) located in the center of the comb backbone (Fig. 6) or at different locations along the comb backbone (Fig. 7). For comparison, we also determined $g(r)$ for a monomer located in the center of a linear polymer backbone. At small distances $r$, neighboring polymers are depleted [$g(r) < 1$] due to steric interactions. Depletion is more significant for comb polymers, consistent with our hypothesis.

We quantify local dilution by determining a characteristic depletion distance $\xi$ that is a proxy for the mesh size of a semidilute unentangled polymer solution. The depletion distance $\xi$ is determined by relating the total number of depleted monomers $n_D = 4\pi \langle \rho \rangle \int_0^\infty r^2 [1 - g(r)] dr$ to an equivalent, fully depleted cavity of size $\xi$, $n_D = 4\pi \langle \rho \rangle \xi^3 / 3$, where $\langle \rho \rangle$ is the average monomer density. The depletion distance $\xi$ is shown in Figure 7 for different branch sizes and concentrations.
density. Equating the two expressions for $n_D$ relates $g(r)$ to $\xi$. For the data shown in Fig. 6, we obtain the ratios $\xi_L/\xi_{C,6} = 0.991$, $\xi_L/\xi_{C,13} = 0.934$, and $\xi_L/\xi_{C,26} = 0.921$ for linear ($L$) to comb ($C$) polymers, where subscripts denote $N_{br} = 6, 13, or 26$. Scaling arguments for semidilute solutions assume that $\xi$ is also a characteristic hydrodynamic screening length, relating polymer concentration ($\phi \propto \xi^{(1-3\nu)/\nu}$) to chain relaxation. Here, the longest polymer relaxation time in semidilute solutions is given by the scaling relation $\tau \propto \phi^{(2\nu-3\nu)/(3\nu-1)}$, where $\nu$ is the solvent quality-dependent Flory exponent.

These relations can be combined to yield the ratio of comb to linear polymer relaxation times $\tau_C/\tau_L \simeq (\xi_L/\xi_C)^{(2-3\nu)/\nu}$. This expression is then used to estimate $\tau_C/\tau_L$, such that $\tau_{C,6}/\tau_L = 0.991$ and $\tau_{C,13}/\tau_L = 0.934$ for $\nu = 1/2$ and $\tau_{C,26}/\tau_L = 0.996$ and $\tau_{C,13}/\tau_L = 0.973$ for $\nu = 3/5$. For longer branches ($N_{br} = 26$), $\tau_{C,26}/\tau_L = 0.921$ for $\nu = 1/2$ and $\tau_{C,26}/\tau_L = 0.971$ for $\nu = 3/5$. Overall, these results are in reasonable agreement with the $\tau_C/\tau_L$ values for the BD simulation results.
shown in Fig. 5 for \( n_{br} = 1 \), supporting our hypothesis that local dilution expedites the relaxation of lightly branched comb polymers compared to linear polymers in semidilute solutions.

Together, these results show that comb-shaped polymers in semidilute solutions exhibit a local dilution effect, such that a small number of branches induces steric repulsion of nearby chains. For chains with long-range HI, local dilution results in more rapid relaxation. This is the predominant effect for relatively low branching density, as increasingly more branches contribute less to steric repulsion and only incur an added frictional drag.

In summary, these results highlight an unexpected trend in the relaxation behavior of comb polymers in semidilute solutions. Our results show that the dynamic behavior of nonlinear polymer chains in semidilute solutions is qualitatively different compared to dilute solutions or melts. In semidilute solutions, a complex interplay between nonlinear polymer architecture, steric interactions, and hydrodynamics results in surprising relaxation behavior that is not captured by existing theories. From this view, our work motivates the need to develop new descriptions for branched polymer dynamics, focusing on the role of concentration, molecular weight, and polymer architecture. Such interactions become increasingly complex as polymer concentration approaches the critical entanglement concentration \([48]\), where understanding topological constraints in the presence of hydrodynamics will be critical to advance new theories for polymer solution dynamics.

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